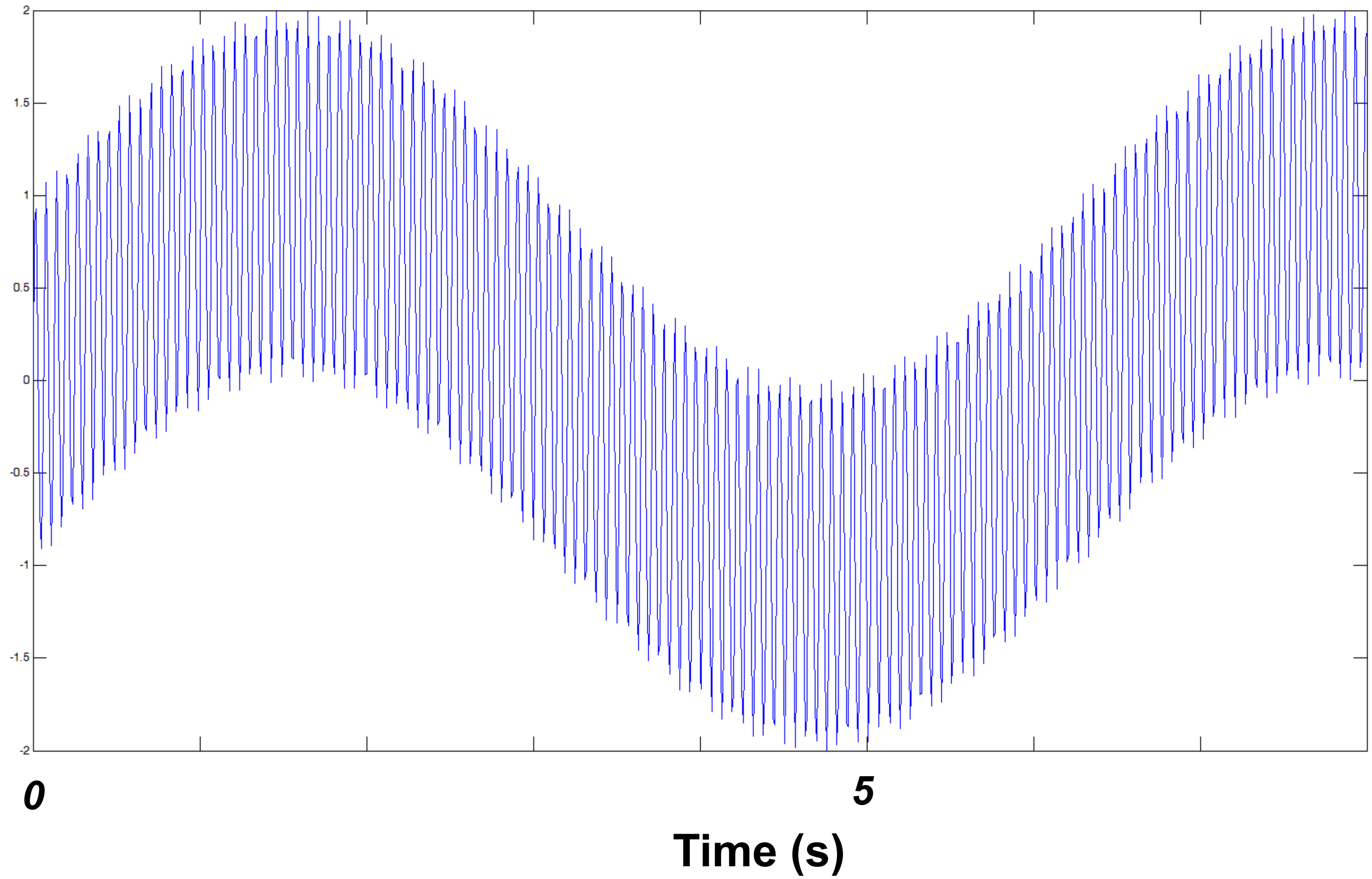
Analog Filtering

Why do we need filters?



- Filters are used to modify the measured signal by removing unwanted signals, i.e. noise.

Noise

- Any unwanted signal that corrupts the signal of interest.
- Common types of noise are:
  - Extraneous noise
  - Thermal noise
  - power noise
  - 1/f noise

Extraneous noise

- Transducers will frequently respond to more than one source.
- Which source is regarded as the signal and which source(s) is regarded as noise is dependent upon the application.
- For instance, electrodes placed on the chest will record the electrical activity of both the heart (ECG) and the respiratory muscles (EMG).

Thermal noise

- In conductors of non-zero resistance, thermally excited electron currents flow randomly and sum statistically to produce a noise voltage across the conductor’s end terminals. This effect is called thermal noise. (Also called Johnson-Nyquist noise).
- At frequencies below 100 MHz, the root mean square (rms) amplitude of thermal noise for a resistor is given by Nyquist’s relation:

$$v_{rms} = \sqrt{4KTR\Delta f}$$

Where K is Boltzmann's constant **(**1.38 * 10e-23 joules/K**),** T is absolute temperature, R is resistance, and Δf is system bandwidth

Thermal noise

- For a neural recording electrode, its impedance is 5MOhm, and the system bandwidth is 20KHz. Thus the thermal noise is

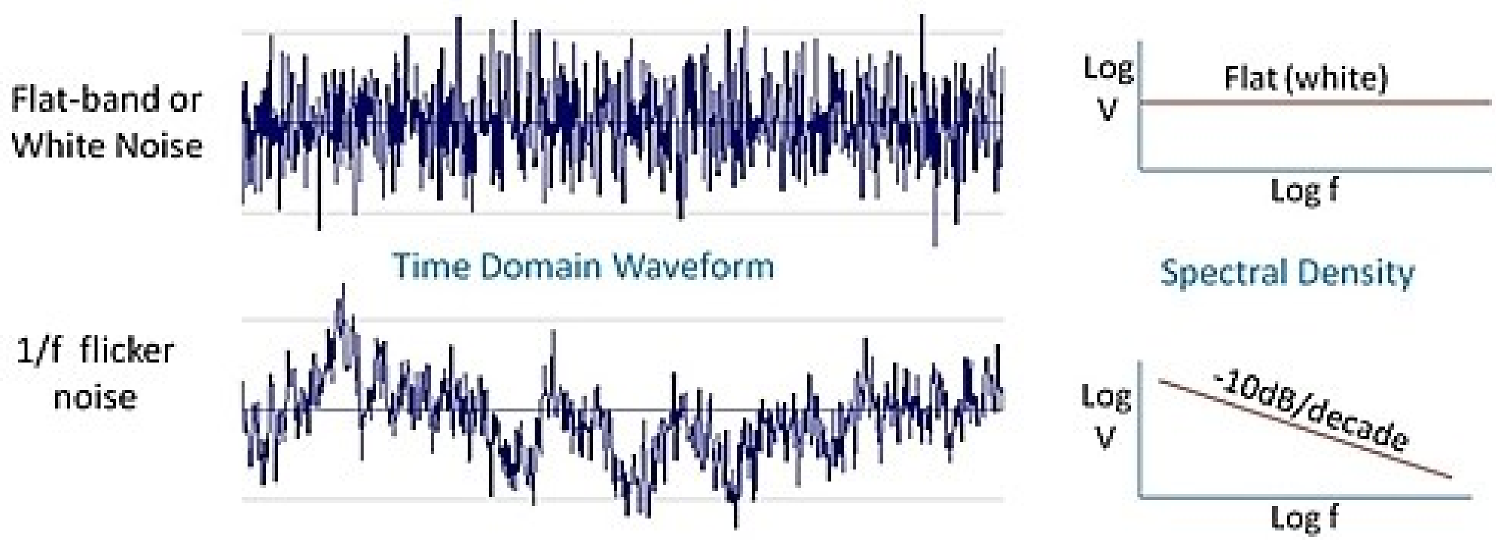
$$v_{rms} = \sqrt{4KTR\Delta f} \approx 40uV$$

Power noise

- Capacitive and/or inductive coupling from AC power can induce 60 Hz noise in wiring and instrumentation.
- Although, power noise has a basic frequency of 60 Hz, nonlinear coupling may give rise to other components at higher (or lower) frequencies.
  - Usually these will occur at integer multiples (60N) or sub-multiples (60/N) of 60Hz.
- Always be suspicious of signals with strong components at such frequencies (i.e. 15, 30, 60, 120, 180 Hz)

1/f noise (pink noise)

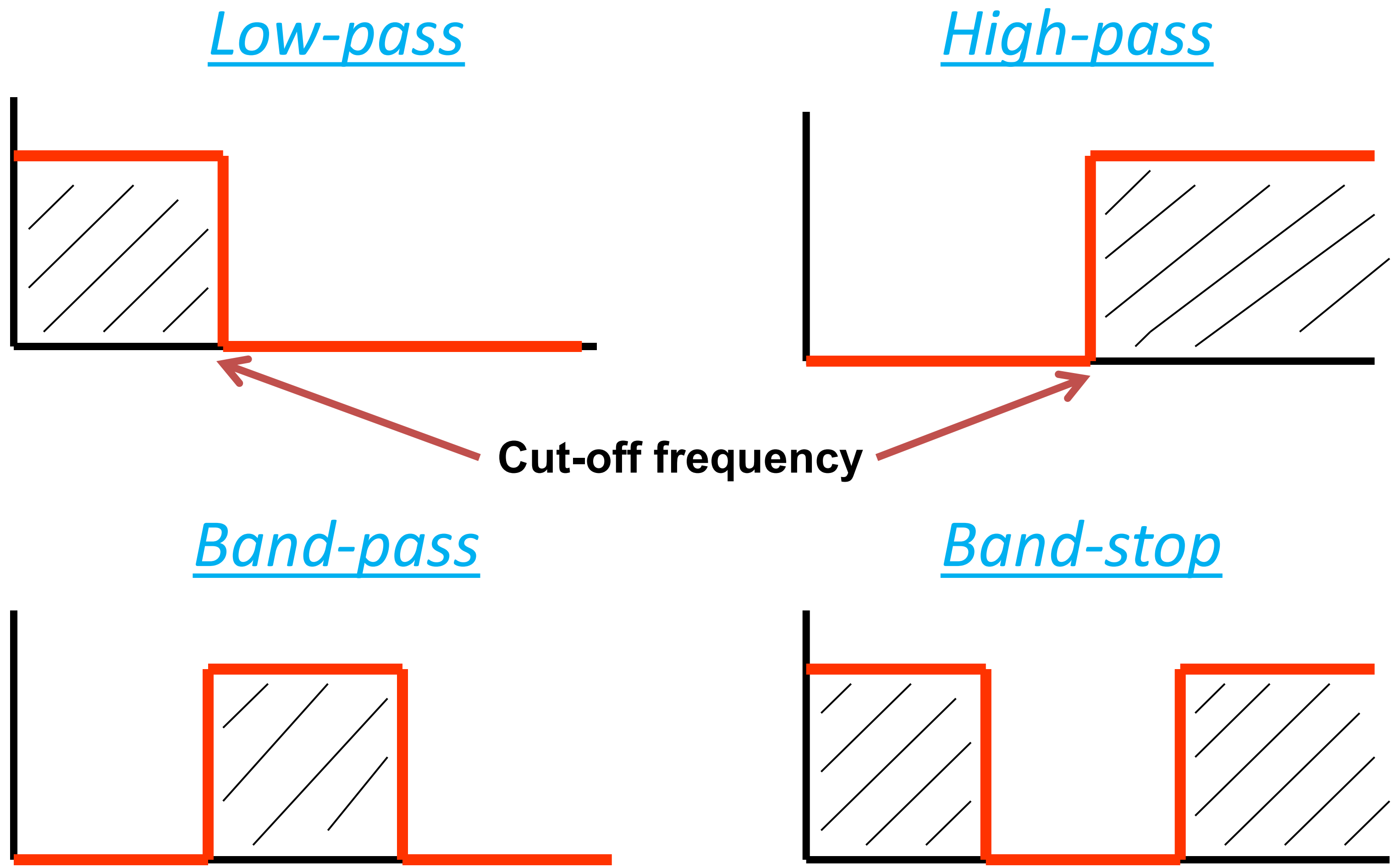
- Caused by fluctuation of resistance of materials
- In contrast to white noise, 1/f noise exhibits a pattern that high frequency noise rides on larger low frequency content.
- Noise amplitude is inversely proportional to the frequency



Analog filters

- Are usually implemented with R, L, and C components and operational amplifiers.
- May be classified as either passive or active
  - A passive filter typically contains only R, L, and C components. It is not necessary that all three be present.
  - An active filter contains, along with R, L, and C components, an energy source, such as an operational amplifier.

Four types of filters



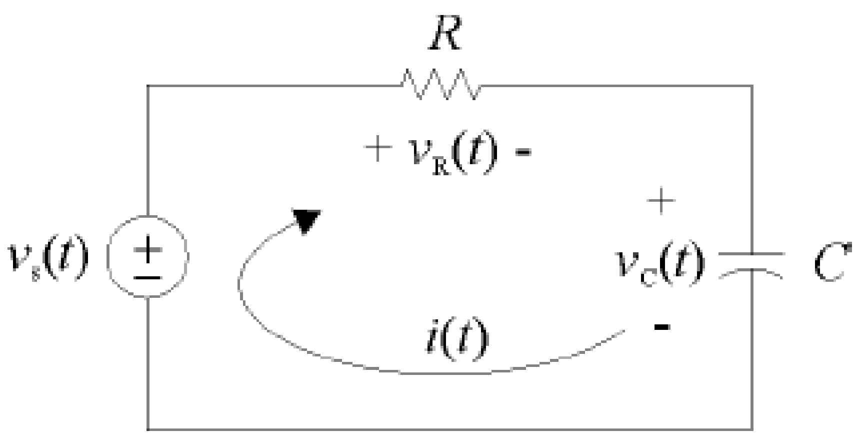
Filter characteristics

- The *cutoff frequency or corner frequency* of a filter is used to describe the transition point from the pass band to the reject band.
- Since this cannot occur instantaneously, it is usually defined as the point where the filter output has decreased to 0.707 of the value in the pass band.
- This corresponds to a gain of -3dB and so the cutoff frequency is sometimes referred to as the - 3dB point.

Filter characteristics

- The *roll-off r*efers to rate at which the filter attenuates the input after the cutoff point.
- In many cases, the filter gain will decrease linearly on the Bode plot so that the roll off can be specified as the linear slope.
- Thus roll-off may be quoted in terms of dB/decade where a decade refers to a factor of ten in frequency.
- Sometimes roll-off will be quoted in terms of dB/octave where an octave refers to a doubling of frequency.

Low pass filter



- Input signal:

$$v_s(t) = \sin(t) + \sin(100t)$$

- What is the voltage across the capacitor?

$$V_c(s) = \frac{1}{1+RCs} V_s(s)$$

$$V_s(s) = \frac{1}{s^2+1} + \frac{100}{s^2+100^2}$$

Low-pass filter

$$V_c(s) = \frac{1}{1+RCs}\cdot\left(\frac{1}{s^2+1}+\frac{100}{s^2+100^2}\right)$$

$$= \frac{1}{1+RCs}\cdot\frac{1}{s^2+1}+\frac{1}{1+RCs}\cdot\frac{100}{s^2+100^2}$$

Response to sinusoidal input with a freq. of 1 rad/s

Response to sinusoidal input with a freq. of 100 rad/s

Low-pass filter

- Response to the 1 rad/s sinusoidal signal

$$V_{c1}(s) = \frac{1/RC}{s + 1/RC} \cdot \frac{1}{s^2 + 1}$$

$$L^{-1}[V_{c1}(s)] = \frac{1}{RC}\left(\frac{1}{(\frac{1}{RC})^2 + 1} e^{-\frac{t}{RC}} + \frac{1}{\sqrt{(\frac{1}{RC})^2 + 1}} \sin(t - \theta)\right)$$

$$= \frac{RC}{(RC)^2 + 1} e^{-\frac{t}{RC}} + \frac{1}{\sqrt{(RC)^2 + 1}} \sin(t - \theta)$$

Where: $\theta = \tan^{-1}(RC)$

Transient response

Steady-state response

Low-pass filter

- Response to the 100 rad/s sinusoidal signal

$$V_{c2}(s) = \frac{1/RC}{s + 1/RC} \cdot \frac{100}{s^2 + 100^2}$$

$$L^{-1}[V_{c2}(s)] = \frac{1}{RC}\left(\frac{100}{(\frac{1}{RC})^2 + 100^2} e^{-\frac{t}{RC}} + \frac{1}{\sqrt{(\frac{1}{RC})^2 + 100^2}} \sin(100t - \theta)\right)$$

$$= \frac{100RC}{(100RC)^2 + 1} e^{-\frac{t}{RC}} + \frac{1}{\sqrt{(100RC)^2 + 1}} \sin(100t - \theta)$$

Where: $\theta = \tan^{-1}(100RC)$

Low-pass filter

$$v_s(t) = \sin(t) + \sin(100t)$$

- If RC = 1

$$V_{c1}(t) = \frac{1}{\sqrt{(RC)^2 + 1}} \sin(t - \theta) = 0.707 \cdot \sin(t - \theta)$$

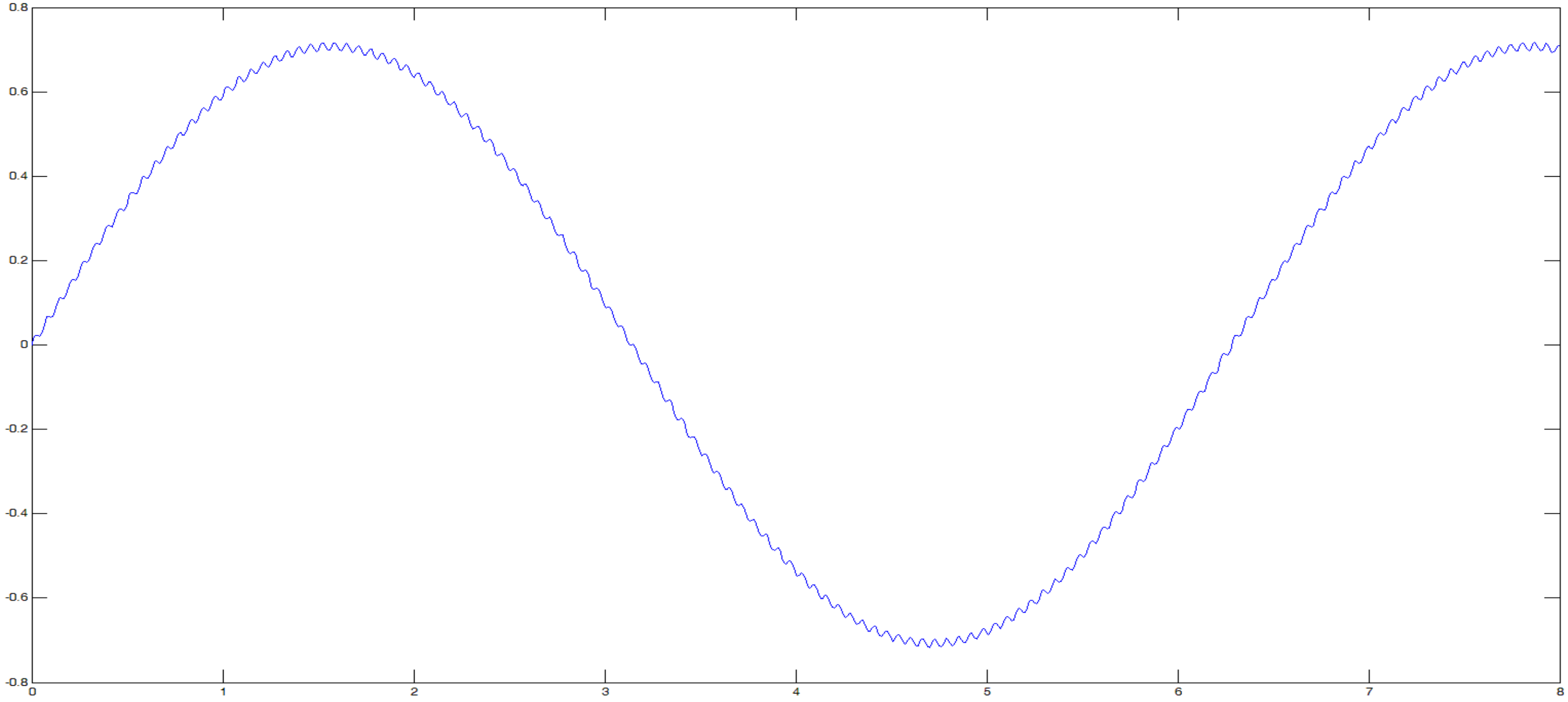
$$V_{c2}(t) = \frac{1}{\sqrt{(100RC)^2 + 1}} \sin(100t - \theta)$$

$$= \frac{1}{\sqrt{10001}} \sin(100t - \theta) \approx 0.01 \cdot \sin(100t - \theta)$$

High frequency signal was attenuated by 99% while the low frequency signal was attenuated by 30%

Low-pass filter

Filtered signal



Low-pass filter

- What if RC = 0.001?

$$V_{c1}(t) = \frac{1}{\sqrt{(RC)^2 + 1}} \sin(t - \theta) \approx \sin(t - \theta)$$

$$V_{c2}(t) = \frac{1}{\sqrt{(100RC)^2 + 1}} \sin(100t - \theta) \approx \sin(100t - \theta)$$

- Neither the high frequency signal nor the low frequency signal was attenuated!
- RC determines the cut-off frequency of the filter.

Low-pass filter

- What if RC = 100?

$$V_{c1}(t) = \frac{1}{\sqrt{(RC)^2 + 1}} \sin(t - \theta) \approx 0.01 \cdot \sin(t - \theta)$$

$$V_{c2}(t) = \frac{1}{\sqrt{(100RC)^2 + 1}} \sin(100t - \theta) \approx 0.0001 \sin(100t - \theta)$$

- Both the high frequency signal and the low frequency signal was attenuated.

Frequency response of a RC circuit

- Response to a sinusoidal signal with angular frequency ω

$$V_c(s) = \frac{1/RC}{s + 1/RC} \cdot \frac{\varpi}{s^2 + \varpi^2}$$

$$L^{-1}[V_c(s)] = \frac{1}{RC}\left(\frac{\varpi}{(\frac{1}{RC})^2 + \varpi^2} e^{-\frac{t}{RC}} + \frac{1}{\sqrt{(\frac{1}{RC})^2 + \varpi^2}} \sin(\varpi t - \theta)\right)$$

$$= \frac{\varpi RC}{(\varpi RC)^2 + 1} e^{-\frac{t}{RC}} + \frac{1}{\sqrt{(\varpi RC)^2 + 1}} \sin(\varpi t - \theta)$$

Where: $\theta = \tan^{-1}(\varpi RC)$

Frequency response of a RC circuit

- Steady state response

$$V_c(t) = \frac{1}{\sqrt{(\varpi RC)^2 + 1}} \sin(\varpi t - \theta)$$

Where: $\theta = \tan^{-1}(\varpi RC)$

- If RC = 1

$$\varpi = 0.01, \quad \frac{1}{\sqrt{(\varpi RC)^2 + 1}} \approx 1$$

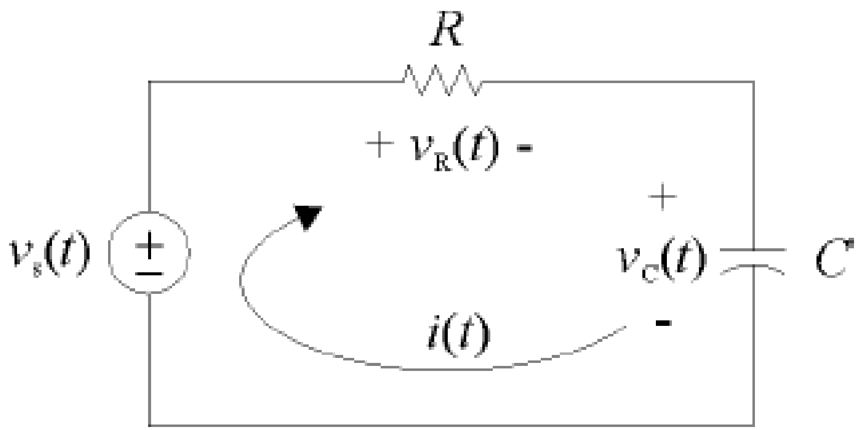
$$\varpi = 0.1, \quad \frac{1}{\sqrt{(\varpi RC)^2 + 1}} \approx 0.995$$

$$\varpi = 1, \quad \frac{1}{\sqrt{(\varpi RC)^2 + 1}} \approx 0.707$$

$$\varpi = 10, \quad \frac{1}{\sqrt{(\varpi RC)^2 + 1}} \approx 0.1$$

$$\varpi = 100, \quad \frac{1}{\sqrt{(\varpi RC)^2 + 1}} \approx 0.01$$

High pass filter



- What if I want to preserve the high frequency signal?
- The voltage across the resistor response to a sinusoidal signal $\sin(\varpi t)$

$$V_R(s) = \frac{RCs}{1+RCs} V_s(s)$$

$$V_R(s) = \frac{RCs}{1+RCs} \cdot \frac{\varpi}{s^2+\varpi^2} = (1 - \frac{1}{1+RCs}) \cdot \frac{\varpi}{s^2+\varpi^2}$$

$$= \frac{\varpi}{s^2+\varpi^2} - \frac{1}{1+RCs} \cdot \frac{\varpi}{s^2+\varpi^2}$$